

Stabilizer ZX Calculus

One of the most important fragments of the ZX calculus:

- All phases are multiples of $\pi/2$.
- Contains “all” the important quantum features from the higher fragments

The Story

- Proven Complete (*Backens*)
 - (2014) Scalar-free
 - (2015) Scalar-aware
- Minimality Program Begins (*Backens, Perdrix, Wang*)
 - (2016) The ruleset gets simplified and only meta-rule is connectivity
 - (2017-2020) All rules except the Bialgebra (!) and red identity removal proven necessary.

So ... what is the status of the Bialgebra and red identity rules???

(S3'R) is Necessary

Let:

$$\left[\begin{array}{c} \text{Spider with } \alpha \text{ and } m \text{ legs} \end{array} \right]^{(S3'R)} := i^{n+m-2} \cdot [Z_{n,m}^\alpha]$$

$$\left[\begin{array}{c} \text{Spider with } \beta \text{ and } m \text{ legs} \end{array} \right]^{(S3'R)} := -1 \cdot [X_{n,m}^\beta]$$

All nontarget rules hold, but...

$$\left[\text{Cap} \right]^{(S3'R)} = \eta \quad \left[\text{Cup} \right]^{(S3'R)} = -\eta$$

Hence, $\text{ZX}_{\text{simp}} \setminus \{(S3'R)\} \not\models (S3'R)$

The Bialgebra is Necessary

Let $V := R^4$ with basis (e_0, e_1, e_2, e_3) , where $R := \mathbb{F}_5[\varepsilon]/(\varepsilon^2)$.

where $\varepsilon \neq 0$, $\varepsilon^2 = 0$ in R .

Our target category is defined by

$$\mathcal{C}_5(n, m) := \text{Hom}_R(V^{\otimes n}, V^{\otimes m}), \quad V^{\otimes 0} := R.$$

$$k \in \mathbb{Z}, \quad t := (1, 3 + 2\varepsilon, 3 + 3\varepsilon, 4).$$

$$\left[\text{Box} \right]^{(B2')} = \frac{1}{2}(H_0 + \varepsilon H_1)$$

$$H_0 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 4 & 1 & 4 \\ 1 & 1 & 4 & 4 \\ 1 & 4 & 4 & 1 \end{pmatrix} \quad H_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 1 \\ 0 & 0 & 1 & 4 \\ 0 & 1 & 4 & 0 \end{pmatrix}$$

$$\left[\text{Spider} \right]^{(B2')} := \sum_{r=0}^3 t_r^k |e_r^{\otimes m}\rangle \langle e_r^{\otimes n}|$$

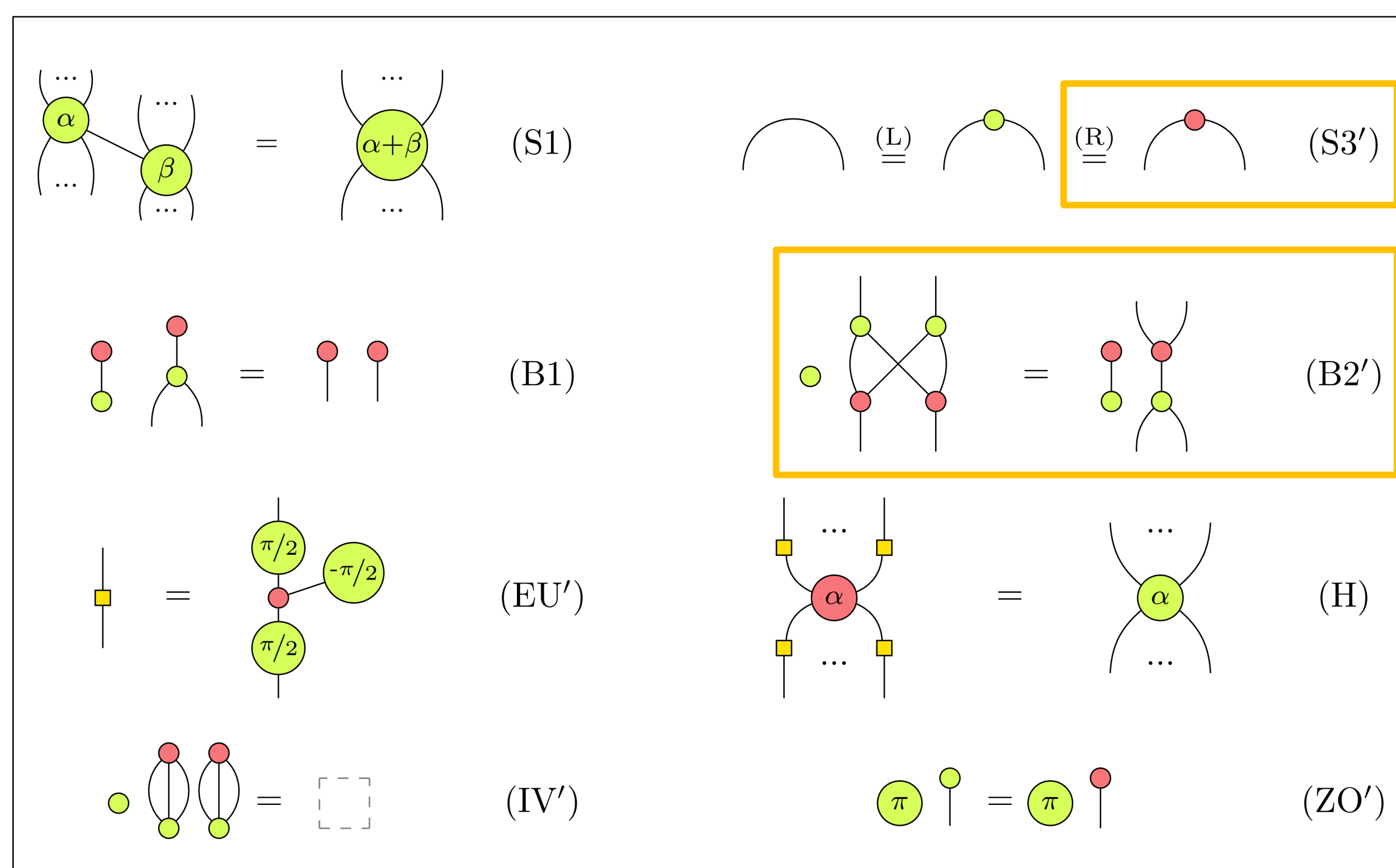
$$\left[\text{Box} \right]^{(B2')} := \left(\left[\text{Box} \right]^{(B2')} \right)^{\otimes m} \circ \left[\text{Spider} \right]^{(B2')} \circ \left(\left[\text{Box} \right]^{(B2')} \right)^{\otimes n}$$

All the nontarget rules hold, but the X and Z spiders are no longer strongly complementary!

(B2') fails by direct computation.

So, $\text{ZX}_{\text{simp}} \setminus \{(B2')\} \not\models (B2')$

The Ruleset



+ “Only connectivity matters”

Minimality of the Stabilizer ZX Calculus
[arXiv:2606.12383 \[quant-ph\], 2026](https://arxiv.org/abs/2606.12383)

Both (S3'R) and (B2') are necessary!